1 Light Cone Coordinates: Definitions, Identities

A four-vector is not bold-faced (e.g. p, k), a three-vector is bold-faced with a vector symbol (e.g. $\vec{\mathbf{p}}$, $\vec{\mathbf{k}}$), and a transverse two-vector is bold-faced without a vector symbol (e.g. $\mathbf{p}$, $\mathbf{k}$). Minkowski four-vectors are written with parentheses, (); light-cone four-vectors with brackets, [].

$$p = (p^0, p^z, \mathbf{p}) = [p^+, p^-, \mathbf{p}]. \tag{1}$$

We will use non-symmetrized lightcone coordinates:

$$p^+ = p^0 + p^z \tag{2}$$

$$p^- = p^0 - p^z \tag{3}$$

$$\mathbf{p} = \mathbf{p}. \tag{4}$$

The inverse transformation is then

$$p^0 = \frac{1}{2}(p^+ + p^-) \tag{5}$$

$$p^z = \frac{1}{2}(p^+ - p^-) \tag{6}$$

$$\mathbf{p} = \mathbf{p}. \tag{7}$$

The Minkowski dot product in lightcone coordinates is:

$$p \cdot k = p^0 k^0 - p^z k^z - \mathbf{p} \cdot \mathbf{k} = \frac{1}{2}(p^+ k^- + p^- k^+) - \mathbf{p} \cdot \mathbf{k}. \tag{8}$$

The length of a vector using lightcone coordinates is then:

$$p \cdot p = p^+ p^- - \mathbf{p} \cdot \mathbf{p}. \tag{9}$$

2 Derivation of WHDG Kinematic Limits

First, see Fig. 1 for notation. For a massless parent parton of momentum P, a massless radiated gluon of momentum k, and a final massless parent parton momentum of p we have that

$$P = (E, E, 0, 0) = [E^+, 0, 0], \tag{10}$$

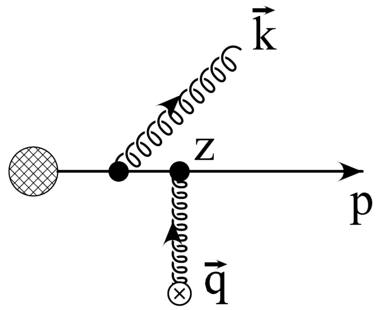

Figure 1: One of the diagrams contributing to the first order in opacity matrix element. $\mathbf{q}$ is the momentum transfer between the parent parton and the in-medium scattering center. $\mathbf{k}$ is the momentum carried off by the radiated gluon. z is the distance from the hard production vertex of the parent parton and the scattering center. Note that the parent parton emerges with momentum P from the blob on the left. Figure adapted from Djordjevic and Gyulassy, Nucl.Phys.A733:265-298, 2004.

where $(,)$ denote the usual 4-momenta, $[,]$ denote light-cone momenta, and we choose the normalization between the two as $E^+ = 2E$. Taking x to be the fraction of ***plus*** momentum carried away by the radiated gluon then

$$k = [xE^+, \frac{\mathbf{k}_\perp^2}{xE^+}, \mathbf{k}_\perp] \tag{11}$$

$$p = [(1-x)E^+, \frac{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2}{(1-x)E^+}, \mathbf{q}_\perp - \mathbf{k}_\perp]. \tag{12}$$

The assumption of eikonality requires that the parent parton continues essentially along its original path. This clearly implies $p^+ \gg p^-$; i.e. that radiating a gluon doesn't make the parent parton go backwards. Similarly we require that the radiated gluon go in the forward direction, $k^+ \gg k^-$; i.e. radiative energy loss does not lead to an energy gain. The first of these conditions implies that ($k_\perp = |\mathbf{k}_\perp|$)

$$(1-x)E^+ \gg |\mathbf{q}_\perp - \mathbf{k}_\perp| \approx k_\perp, \tag{13}$$

where we note that $q_{max} = \sqrt{6ET} \ll k$ for $E \gg T$; the second condition that

$$xE^+ \gg k_\perp. \tag{14}$$

Taking the $k_\perp$ integral cutoff to occur precisely at equality for these conditions leads to

$$
\begin{aligned}
k_{max} &= \min(x, 1-x)E^+ && (15)\\
&= 2\min(x, 1-x)E && (16)\\
&\approx 2x(1-x)E, && (17)
\end{aligned}
$$

where the last line is used in the WHDG implementation for convergence reasons and makes little difference in the final result (see the last TECHQM meeting).

3 On Mass Shell Gluon

First note that Eq. (11) implies that the massless gluon is always on shell,

$$
k \cdot k = xE^+ \frac{\mathbf{k}_\perp^2}{xE^+} - \mathbf{k}_\perp \cdot \mathbf{k}_\perp = 0. \tag{18}
$$

Specifically, Urs claimed that when $k_\perp = k_{max}$ then $\omega = 2xE = 2\omega$. It will be shown clearly below that this is due to naively taking $\omega = k^+/2$. While this is valid when $E^+ \to \infty$ keeping $k_\perp$ fixed, this assumption is not valid at the cutoff $k_\perp \sim xE^+$. Inverting the lightcone transformation back into 4-momenta

$$
\begin{aligned}
k \quad &= \quad (\frac{1}{2}\left(xE^+ + \frac{\mathbf{k}_\perp^2}{xE^+}\right), \frac{1}{2}\left(xE^+ - \frac{\mathbf{k}_\perp^2}{xE^+}\right), \mathbf{k}_\perp) && (19)\\
&\overset{k_\perp \to k_{max}}{\Longrightarrow} \quad (xE^+, 0, xE^+ \frac{\mathbf{k}_\perp}{k_\perp}). && (20)
\end{aligned}
$$

Clearly we have $\omega = k_{max}$ and $k \cdot k = 0$ when $k_\perp = k_{max}$.

Ultimately the matrix element calculation assumes eikonality; i.e. $k_\perp \ll xE^+$. WHDG imposes this onto the $k_\perp$ integration by cutting off when equality between the quantities is reached, i.e. $k_{max} = xE^+ = 2xE$. There is clearly at least an $\mathcal{O}(1)$ multiplicative uncertainty in the determination of k_{max} that will result, at the current level of approximation and as described in the note, in a large systematic theoretical uncertainty in the calculation of any physical observable.